Chapter 11
Heat Exchangers

Heat Exchangers:
Design Considerations

Chapter 11

Sections 11.1 through 11.3

Heat Exchangers

"The process of heat exchange between two fluids that are at different temperatures and separated by a solid wall occurs in many engineering applications. The device used to implement this exchange is termed a heat exchanger, and specific application may be found in space heating and air-conditioning, power production, waste heat recovery, and chemical processing" [Incropera, 632]

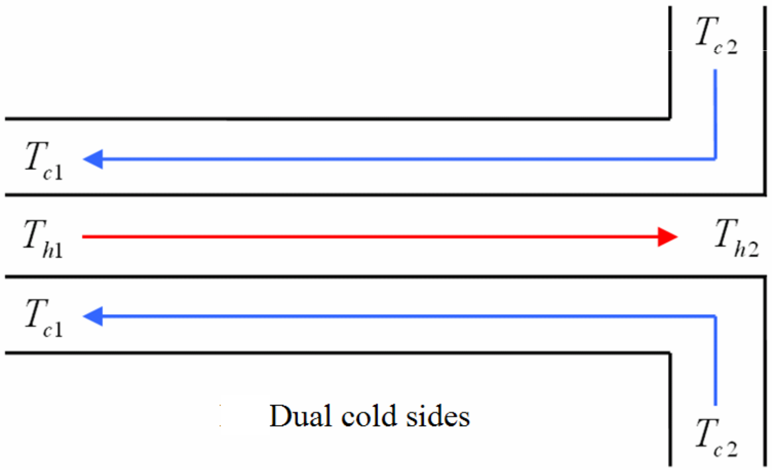
Definition: Device providing heat exchange between two fluids that are at different temperatures and separated by a solid surface.

For a heat exchanger you need a high temperature flow and a low temperature flow.

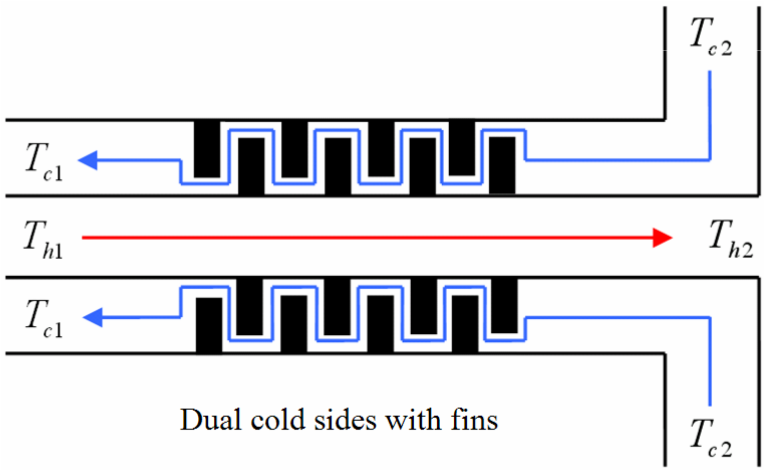
A simple heat exchanger could just be a flat plate:

Heat Exchangers

More complicated heat exchangers could have dual cooling intakes and fins:



Dual cold sides

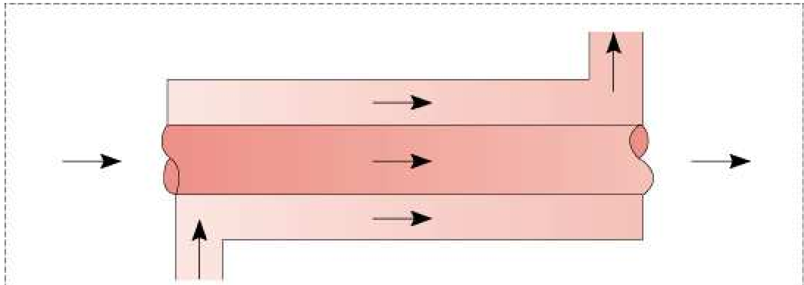


Dual cold sides with fins

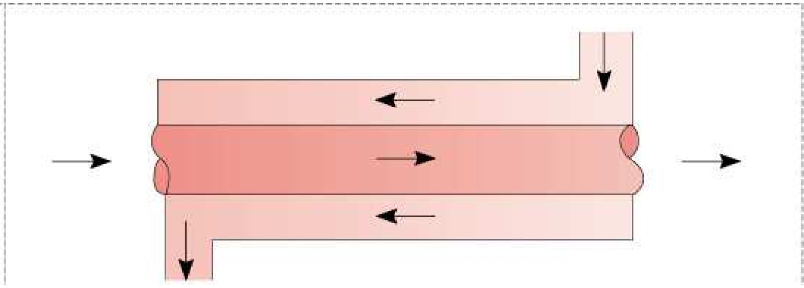
Heat Exchanger Types

Heat exchangers are ubiquitous to energy conversion and utilization. They involve heat exchange between two fluids separated by a solid and encompass a wide range of flow configurations.

- Concentric-Tube Heat Exchangers



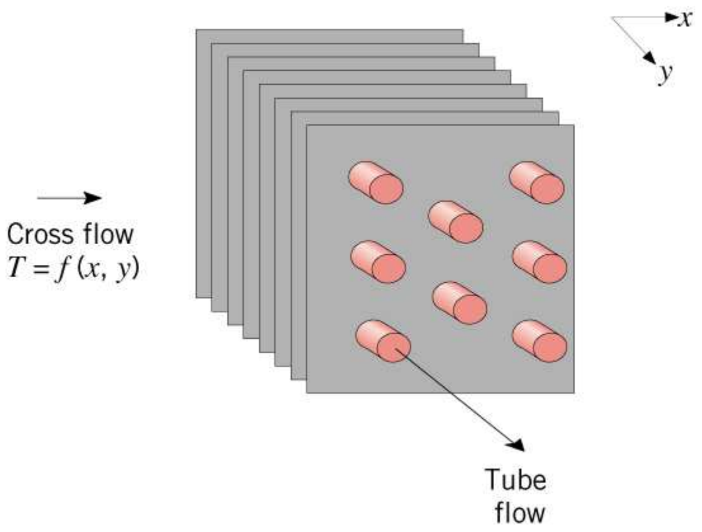
Parallel Flow



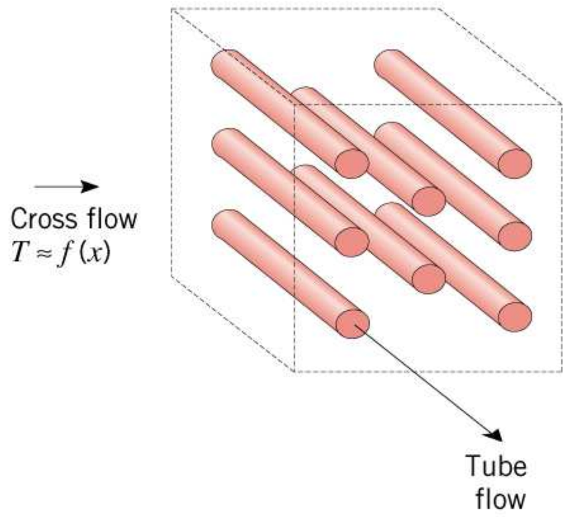
Counterflow

- ➢ Simplest configuration.
- ➢ Superior performance associated with counter flow.

- Cross-flow Heat Exchangers



Finned-Both Fluids Unmixed

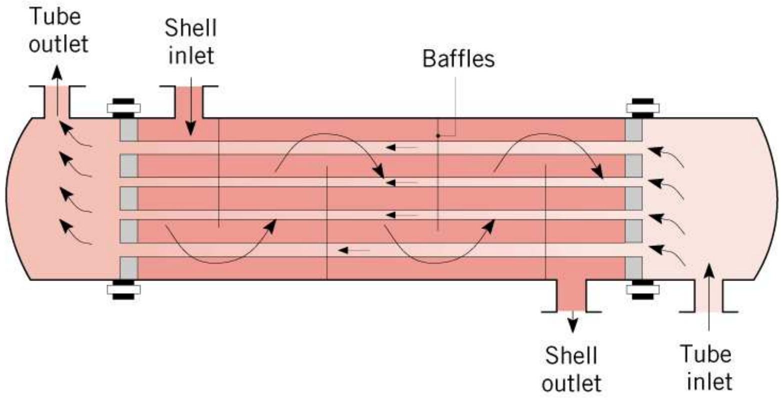


Unfinned-One Fluid Mixed the Other Unmixed

- For cross-flow over the tubes, fluid motion, and hence mixing, in the transverse direction (y) is prevented for the finned tubes, but occurs for the unfinned condition.

- Heat exchanger performance is influenced by mixing.

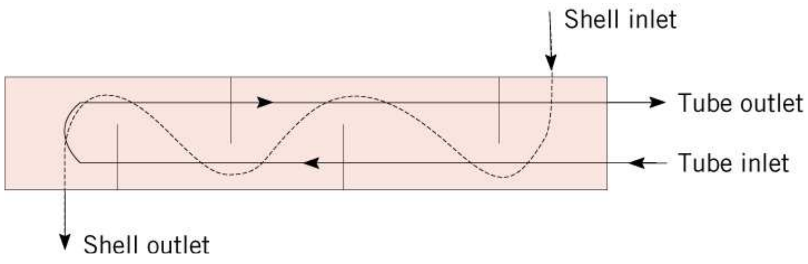
- Shell-and-Tube Heat Exchangers



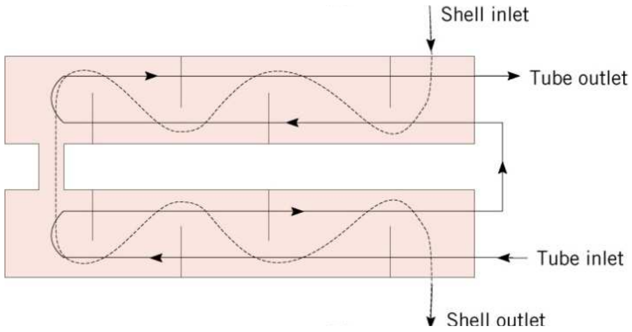
One Shell Pass and One Tube Pass

➢ Baffles are used to establish a cross-flow and to induce turbulent mixing of the shell-side fluid, both of which enhance convection.

➢ The number of tube and shell passes may be varied, e.g.:

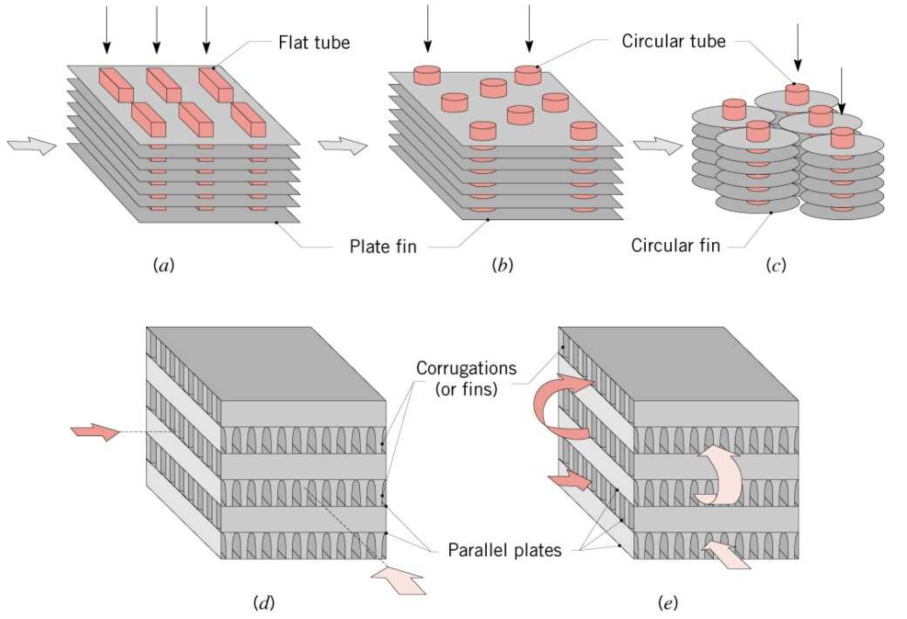


One Shell Pass,
Two Tube Passes



Two Shell Passes,
Four Tube Passes

- Compact Heat Exchangers
  - Widely used to achieve large heat rates per unit volume, particularly when one or both fluids is a gas.
  - Characterized by large heat transfer surface areas per unit volume, small flow passages, and laminar flow.



(a) Fin-tube (flat tubes, continuous plate fins)
(b) Fin-tube (circular tubes, continuous plate fins)
(c) Fin-tube (circular tubes, circular fins)
(d) Plate-fin (single pass)
(e) Plate-fin (multipass)

Overall Heat Transfer Coefficient (U)

Define variables: $T_{h,m}$ = mean high temperature

$T_{c,m}$ = mean cold temperature

A simple way to find $T_{h,m}$ would be to find $\frac{T_{h,2} + T_{h,1}}{2}$, but this assumes that the temperature of the fluid is linear throughout the heat exchanger.

There are two heat exchanger conditions:

1) Cold and hot fluids flow in opposite directions (counterflow)
2) Cold and hot fluids flow in the same direction (parallel flow)

$$q = UA(T_{h,m} - T_{c,m}) = UA\Delta T_m$$

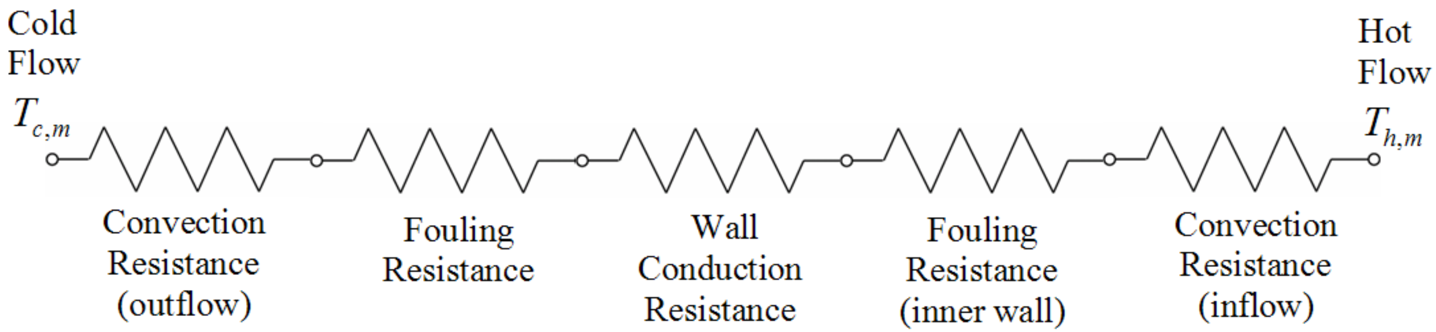
We will analyze this equation by breaking it down into two parts: (UA) and (ΔT_m)

Overall Heat Transfer Coefficient (U)

1) UA

UA ≡ Overall Thermal Resistance

$$q = UA\Delta T_m = \frac{\Delta T_m}{1/UA}$$



$$\frac{1}{UA} = \frac{1}{(hA)_c} + \frac{R''_{f,c}}{A} + R_{wall} + \frac{R''_{f,h}}{A} + \frac{1}{(hA)_h}$$

Overall Heat Transfer Coefficient (U)

1) UA (continued)

$$R_{wall} \begin{cases} \dfrac{L}{k_w A} & \text{(plane wall)} \\ \dfrac{\ln\left(r_2/r_1\right)}{2\pi l K} & \text{(cylinder wall)} \end{cases}$$

where $R''_{f,c}$ = fouling factor (given in Table 11.1) $\left[m^2 K/W\right]$

These equations are for heat exchangers with no fins. If fins are involved, we have a special case.

Specific Case: Finned Exchanger

-in the case of a finned exchanger, the following equation for overall thermal resistance is used.

$$\frac{1}{UA} = \frac{1}{hA\eta_o} + \frac{R''_{f,c}}{A\eta_o} + R_{wall} + \frac{R''_{f,h}}{A\eta_o} + \frac{1}{hA\eta_o}$$

Overall Heat Transfer Coefficient (U)

1) UA (continued)

where η_o ≡ overall surface efficiency or effectiveness of a finned surface

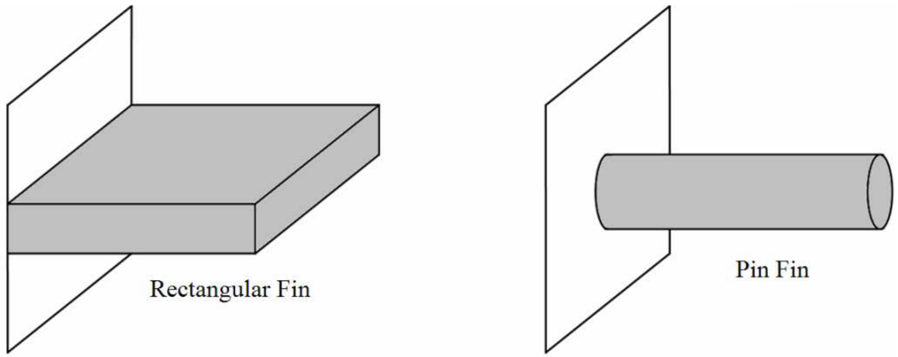
$$q = \eta_o hA(T_b - T_\infty)$$

T_b = base temperature of fin

A= total surface area of fin (fin + base area)

$$\eta_o = 1 - \frac{A_f}{A}(1 - \eta_f)$$

η_f =effectiveness of single fin



Overall Heat Transfer Coefficient (U)

1) UA (continued)

$$\eta_f = \frac{\tanh(mL)}{mL}$$

$$m = \left[2h/kt\right]^{1/2}$$

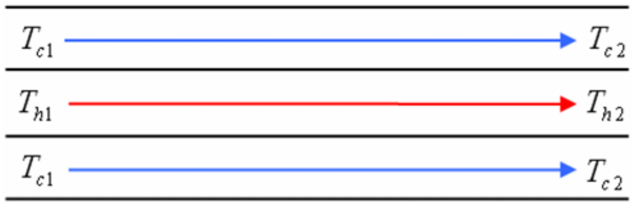
where t = fin thickness

2) How to find T_m

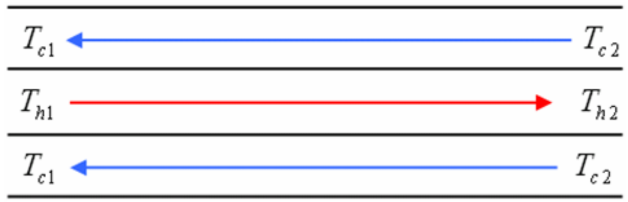
a) parallel flow exchangers: $$\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\Delta T_2 / \Delta T_1\right)}$$

b) counterflow exchangers: $$\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\Delta T_2 / \Delta T_1\right)}$$

where $\Delta T_2 = T_{h2} - T_{c2}$ and $\Delta T_1 = T_{h1} - T_{c1}$



Parallel Flow



Counterflow

Overall Heat Transfer Coefficient

- An essential requirement for heat exchanger design or performance calculations.

- Contributing factors include convection and conduction associated with the two fluids and the intermediate solid, as well as the potential use of fins on both sides and the effects of time-dependent surface fouling.

- With subscripts *c* and *h* used to designate the *hot* and *cold* fluids, respectively, the most general expression for the overall coefficient is:

$$\frac{1}{UA} = \frac{1}{(UA)_c} = \frac{1}{(UA)_h}$$

$$= \frac{1}{(\eta_o hA)_c} + \frac{R''_{f,c}}{(\eta_o A)_c} + R_w + \frac{R''_{f,h}}{(\eta_o A)_h} + \frac{1}{(\eta_o hA)_h}$$

- $R''_f \rightarrow$ Fouling factor for a unit surface area $(\mathrm{m}^2 \cdot \mathrm{K/W})$
  $\rightarrow$ Table 11.1

- $R_w \rightarrow$ Wall conduction resistance (K/W)

- $\eta_o \rightarrow$ Overall surface efficiency of fin array (Section 3.6.5)

$$\eta_{o,c\ or\ h} = \left(1 - \frac{A_f}{A}\left(1 - \eta_f\right)\right)_{c\ or\ h}$$

$A = A_t \rightarrow$ total surface area (fins and exposed base)
$A_f \rightarrow$ surface area of fins only

Assuming an adiabatic tip, the fin efficiency is

$$\eta_{f,c\,or\,h} = \left(\frac{\tanh(mL)}{mL}\right)_{c\ or\ h}$$

$$m_{c\ or\ h} = \left(2U_p / k_w t\right)_{c\ or\ h}$$

$$U_{p,c\ or\ h} = \left(\frac{h}{1 + hR''_f}\right)_{c\ or\ h} \rightarrow \text{partial overall coefficient}$$

A Methodology for Heat Exchanger Design Calculations

- The Log Mean Temperature Difference (LMTD) Method -

- A form of Newton's Law of Cooling may be applied to heat exchangers by using a log-mean value of the temperature difference between the two fluids:

$$q = U\,A\,\Delta T_{1m}$$

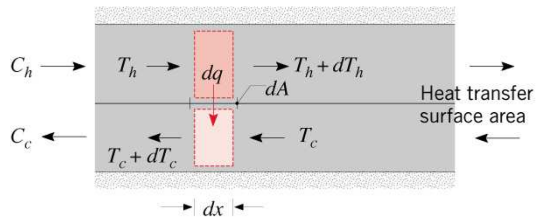
$$\Delta T_{1m} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

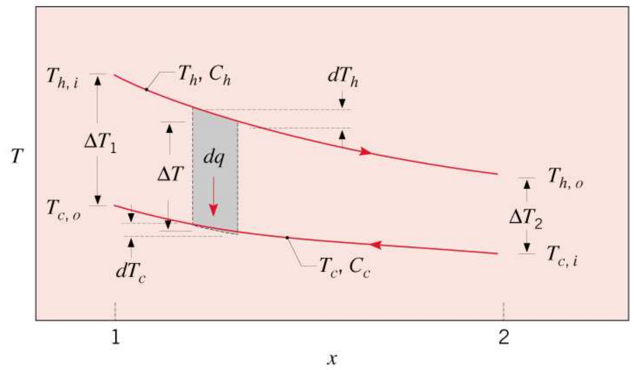
Evaluation of ΔT_1 and ΔT_2 depends on the heat exchanger type.

- Counter-Flow Heat Exchanger:

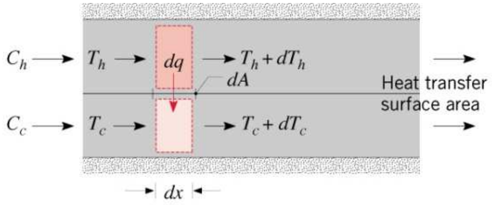
$$\Delta T_1 \equiv T_{h,1} - T_{c,1} = T_{h,i} - T_{c,o}$$

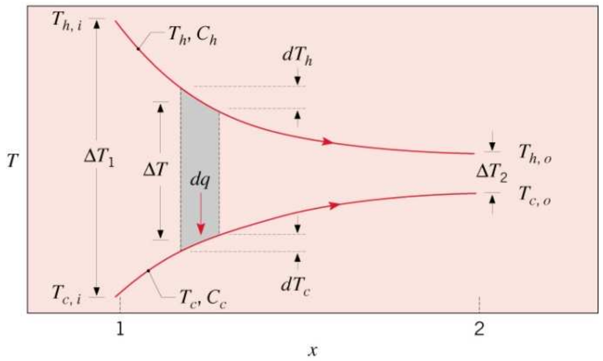
$$\Delta T_2 \equiv T_{h,2} - T_{c,2} = T_{h,o} - T_{c,i}$$





- Parallel-Flow Heat Exchanger:





$$\Delta T_1 \equiv T_{h,1} - T_{c,1} = T_{h,i} - T_{c,i}$$

$$\Delta T_2 \equiv T_{h,2} - T_{c,2} = T_{h,o} - T_{c,o}$$

- Note that $T_{c,o}$ can not exceed $T_{h,o}$ for a PF HX, but can do so for a CF HX.
- For equivalent values of UA and inlet temperatures,

$$\Delta T_{1m,CF} > \Delta T_{1m,PF}$$

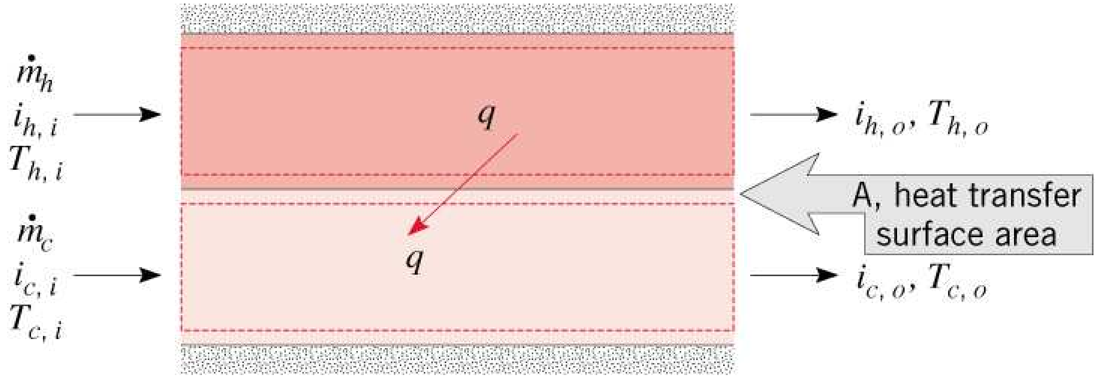
- Shell-and-Tube and Cross-Flow Heat Exchangers:

$$\Delta T_{1m} = F\ \Delta T_{1m,CF}$$

$$F \rightarrow \text{Figures 11S.1 - 11S.4}$$

Overall Energy Balance

- Application to the *hot (h)* and *cold (c)* fluids:



- Assume negligible heat transfer between the exchanger and its surroundings and negligible potential and kinetic energy changes for each fluid.

$$q = \dot{m}_h \left(i_{h,i} - i_{h,o} \right)$$

$$q = \dot{m}_c \left(i_{c,o} - i_{c,i} \right)$$

$i \rightarrow$ fluid enthalpy

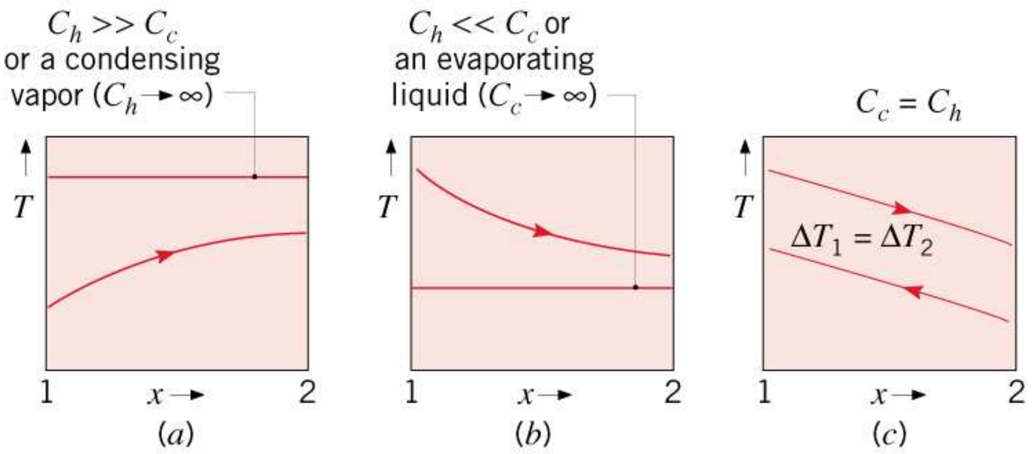
- Assuming no *l/v* phase change and constant specific heats,

$$q = \dot{m}_h c_{p,h} \left(T_{h,i} - T_{h,o} \right) = C_h \left(T_{h,i} - T_{h,o} \right)$$

$$q = \dot{m}_c c_{p,c} \left(T_{c,o} - T_{c,i} \right) = C_c \left(T_{c,o} - T_{c,i} \right)$$

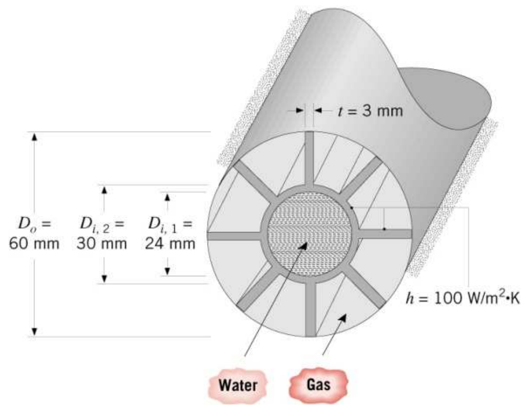
$C_h, C_c \rightarrow$ Heat capacity rates

Special Operating Conditions



- Case (a): $C_h >> C_c$ or h is a condensing vapor $(C_h \rightarrow \infty)$.
  - Negligible or no change in $T_h \left(T_{h,o} = T_{h,i}\right)$.
- Case (b): $C_c >> C_h$ or c is an evaporating liquid $(C_c \rightarrow \infty)$.
  - Negligible or no change in $T_c \left(T_{c,o} = T_{c,i}\right)$.
- Case (c): $C_h = C_c$.
  - $\Delta T_1 = \Delta T_2 = \Delta T_{1m}$

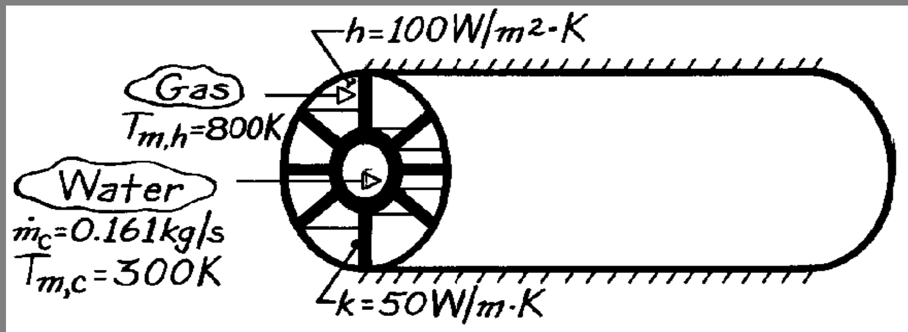
Problem 11.5: Determination of heat transfer per unit length for heat recovery device involving hot flue gases and water.



KNOWN: Geometry of finned, annular heat exchanger. Gas-side temperature and convection coefficient. Water-side flowrate and temperature.

FIND: Heat rate per unit length.

SCHEMATIC:



D_o = 60 mm
$D_{i,1}$ = 24 mm
$D_{i,2}$ = 30 mm
t = 3 mm = 0.003m
L = (60-30)/2 mm = 0

ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional conduction in strut, (4) Adiabatic outer surface conditions, (5) Negligible gas-side radiation, (6) Fully-developed internal flow, (7) Negligible fouling.

PROPERTIES: *Table A-6*, Water (300 K): k = 0.613 W/m·K, Pr = 5.83, $\mu = 855 \times 10^{-6}$ N·s/m^2.

ANALYSIS: The heat rate is

$$q = (UA)_c \left(T_{m,h} - T_{m,c}\right)$$

where

$$1/(UA)_c = 1/(hA)_c + R_w + 1/(\eta_o hA)_h$$

$$R_w = \frac{\ln\left(D_{i,2}/D_{i,1}\right)}{2\pi kL} = \frac{\ln(30/24)}{2\pi(50\ \text{W/m}\cdot\text{K})\,1\text{m}} = 7.10 \times 10^{-4}\ \text{K/W}.$$

With

$$Re_D = \frac{4\dot{m}}{\pi D_{i,1}\mu} = \frac{4 \times 0.161\ kg/s}{\pi(0.024m)855 \times 10^{-6}\ N \cdot s/m^2} = 9990$$

the internal flow is turbulent and the Dittus-Boelter correlation gives

$$h_c = (k/D_{i,1})0.023\,Re_D^{4/5}\,Pr^{0.4} = \left(\frac{0.613\ W/m \cdot K}{0.024m}\right)0.023(9990)^{4/5}(5.83)^{0.4} = 1883\ W/m^2 \cdot K$$

$$(hA)_c^{-1} = \left(1883\ W/m^2 \cdot K \times \pi \times 0.024m \times 1m\right)^{-1} = 7.043 \times 10^{-3}\ K/W.$$

The overall fin efficiency is

$$\eta_o = 1 - (A_f/A)(1-\eta_f)$$

$$A_f = 8 \times 2(L \cdot w) = 8 \times 2(0.015m \times 1m) = 0.24m^2$$

$$A = A_f + (\pi D_{i,2} - 8t)w = 0.24m^2 + (\pi \times 0.03m - 8 \times 0.003m) = 0.31m^2.$$

From Eq. 11.4,

$$\eta_f = \frac{\tanh(mL)}{mL}$$

where

$$m = [2h/kt]^{1/2} = \left[2 \times 100\ W/m^2 \cdot K / 50\ W/m \cdot K (0.003m)\right]^{1/2} = 36.5\,m^{-1}$$

$$mL = (2h/kt)^{1/2} L = 36.5\,m^{-1} \times 0.015m = 0.55$$

$$\tanh\left[(2h/kt)^{1/2} L\right] = 0.499.$$

Hence

$$\eta_f = 0.499/0.55 = 0.911$$

$$\eta_o = 1 - (A_f/A)(1-\eta_f) = 1 - (0.24/0.31)(1-0.911) = 0.931$$

$$(\eta_o hA)_h^{-1} = \left(0.931 \times 100\ W/m^2 \cdot K \times 0.31m^2\right)^{-1} = 0.0347\ K/W.$$

It follows that

$$(UA)_c^{-1} = \left(7.043 \times 10^{-3} + 7.1 \times 10^{-4} + 0.0347\right) K/W$$

$$(UA)_c = 23.6\ W/K$$

and

$$q = 23.6\ W/K\,(800-300)\,K = 11{,}800\ W \qquad <$$

for a 1m long section.

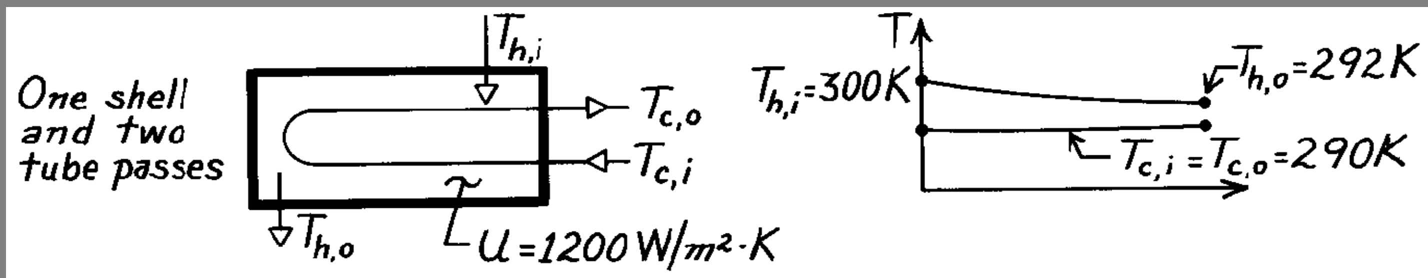
COMMENTS: (1) The gas-side resistance is substantially decreased by using the fins $\left(A'_f >> \pi D_{i,2} \right)$ and q is increased.

(2) Heat transfer enhancement by the fins could be increased further by using a material of larger k, but material selection would be limited by the large value of $T_{m,h}$.

Problem 11S.8: Design of a two-pass, shell-and-tube heat exchanger to supply vapor for the turbine of an ocean thermal energy conversion system based on a standard (Rankine) power cycle. The power cycle is to generate 2 MW_e at an efficiency of 3%. Ocean water enters the tubes of the exchanger at 300K, and its desired outlet temperature is 292K. The working fluid of the power cycle is evaporated in the tubes of the exchanger at its phase change temperature of 290K, and the overall heat transfer coefficient is known.

FIND: (a) Evaporator area, (b) Water flow rate.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Constant properties.

PROPERTIES: *Table A-6*, Water ($\overline{T}_m$ = 296 K): c_p = 4181 J/kg·K.

ANALYSIS: (a) The efficiency is

$$\eta = \frac{\dot{W}}{q} = \frac{2\,\text{MW}}{q} = 0.03.$$

Hence the required heat transfer rate is

$$q = \frac{2\,\text{MW}}{0.03} = 66.7\,\text{MW}.$$

Also

$$\Delta T_{\ell m,CF} = \frac{(300-290)-(292-290)°C}{\ell n\,\frac{300-290}{292-290}} = 5°C$$

and, with P = 0 and R = ∞, from Fig. 11S.1 it follows that F = 1. Hence

$$A = \frac{q}{U\,F\,\Delta T_{\ell m,CF}} = \frac{6.67\times10^{7}\ \text{W}}{1200\ \text{W/m}^2\cdot\text{K}\times1\times5°C}$$

$$A = 11{,}100\,\text{m}^2.$$

b) The water flow rate through the evaporator is

$$\dot{m}_h = \frac{q}{c_{p,h}\left(T_{h,i} - T_{h,o}\right)} = \frac{6.67 \times 10^7 \text{ W}}{4181 \text{ J/kg} \cdot \text{K}\left(300 - 292\right)}$$

$$\dot{m}_h = 1994 \text{ kg/s.}$$

COMMENTS: (1) The required heat exchanger size is enormous due to the small temperature differences involved,.

(2) The concept was considered during the *energy crisis* of the mid 1970s but has not since been implemented

Heat Exchangers: The Effectiveness – NTU Method

Chapter 11

Sections 11.4 through 11.7

General Considerations

- Computational Features/Limitations of the LMTD Method:

  - The LMTD method may be applied to design problems for which the fluid flow rates and inlet temperatures, as well as a desired outlet temperature, are prescribed. For a specified HX type, the required size (surface area), as well as the other outlet temperature, are readily determined.

  - If the LMTD method is used in performance calculations for which both outlet temperatures must be determined from knowledge of the inlet temperatures, the solution procedure is iterative.

  - For both design and performance calculations, the effectiveness-NTU method may be used without iteration.

Definitions

- Heat exchanger effectiveness, ε :

$$\varepsilon = \frac{q}{q_{\max}}$$

$$0 \le \varepsilon \le 1$$

- Maximum possible heat rate:

$$q_{\max} = C_{\min}\left(T_{h,i} - T_{c,i}\right)$$

$$C_{\min} = \begin{cases} C_h \text{ if } C_h < C_c \\ \text{or} \\ C_c \text{ if } C_c < C_h \end{cases}$$

- ➢ Will the fluid characterized by C_{min} or C_{max} experience the largest possible temperature change in transit through the HX?

- ➢ Why is C_{min} and not C_{max} used in the definition of q_{max}?

- Number of Transfer Units, *NTU*

$$NTU \equiv \frac{UA}{C_{\min}}$$

  - A dimensionless parameter whose magnitude influences HX performance:

$$q \uparrow \text{ with } \uparrow NTU$$

Heat Exchanger Relations

- $$\begin{cases} q = \dot{m}_h \left(i_{h,i} - i_{h,o} \right) \\ or \\ q = C_h \left(T_{h,i} - T_{h,o} \right) \end{cases}$$
- $$\begin{cases} q = \dot{m}_c \left(i_{c,o} - i_{c,i} \right) \\ or \\ q = C_c \left(T_{c,o} - T_{c,i} \right) \end{cases}$$
- $$q = \varepsilon C_{\min} \left(T_{h,i} - T_{c,i} \right)$$
- Performance Calculations:
  - $$\varepsilon = f\left(NTU, \underbrace{C_{\min} / C_{\max}}_{C_r} \right)$$
  - Relations $\rightarrow$ Table 11.3 or Figs. 11.10 - 11.15

- Design Calculations:
  - $NTU = f\left(\varepsilon, C_{\min} / C_{\max}\right)$
  - Relations $\rightarrow$ Table 11.4 or Figs. 11.10 - 11.15

- For all heat exchangers,

$$\varepsilon \uparrow \text{ with } \downarrow C_r$$

- For $C_r = 0$, a single $\varepsilon - NTU$ relation applies to all HX types.

$$\varepsilon = 1 - \exp\left(-NTU\right)$$

or

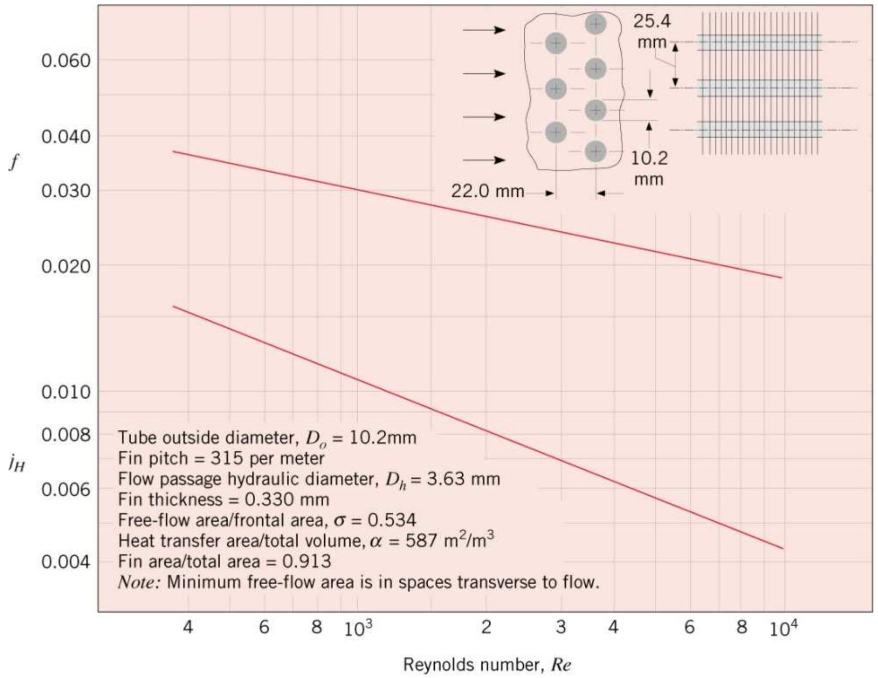
$$NTU = -\ln\left(1-\varepsilon\right)$$

Compact Heat Exchangers

- Analysis based on $\varepsilon - NTU$ method

- Convection (and friction) coefficients have been determined for selected HX cores by Kays and London [5] Proprietary data have been obtained by manufacturers of many other core configurations.

- Results for a circular tube-continuous fin HX core:

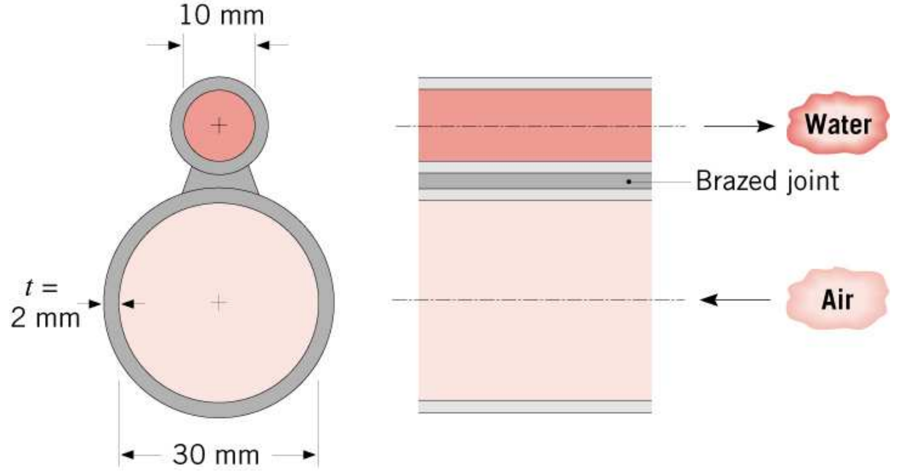


$$j_h = St\,\mathrm{Pr}^{2/3}$$

$$St = h / Gc_p$$

$$G = \rho V_{max}$$

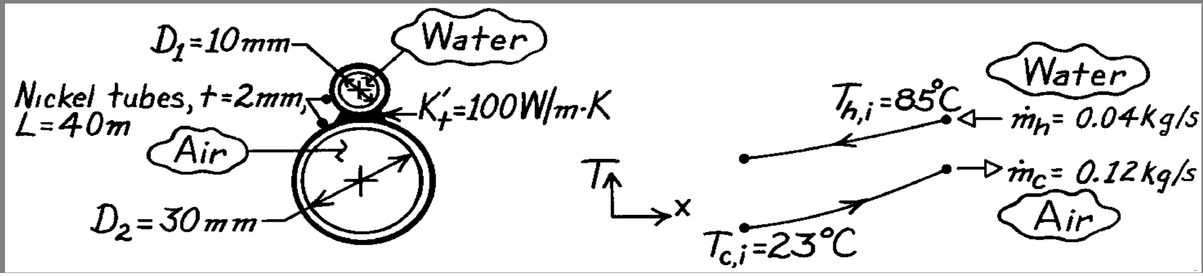
Problem 11.28: Use of twin-tube (brazed) heat exchanger to heat air by extracting energy from a hot water supply.



KNOWN: Counterflow heat exchanger formed by two brazed tubes with prescribed hot and cold fluid inlet temperatures and flow rates.

FIND: Outlet temperature of the air.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible loss/gain from tubes to surroundings, (2) Negligible changes in kinetic and potential energy, (3) Flow in tubes is fully developed since L/D_h = 40 m/0.030m = 1333.

PROPERTIES: *Table A-6*, Water ($\overline{T}_h$ = 335 K): $c_h = c_{p,h}$ = 4186 J/kg·K, μ = 453 × 10^{-6} N·s/m^2, k = 0.656 W/m·K, Pr = 2.88; *Table A-4*, Air (300 K): $c_c = c_{p,c}$ = 1007 J/kg·K, μ = 184.6 × 10^{-7} N·s/m^2, k = 0.0263 W/m·K, Pr = 0.707; *Table A-1*, Nickel ($\overline{T}$ = (23 + 85)°C/2 = 327 K): k = 88 W/m·K.

ANALYSIS: Using the NTU - ε method, from Eq. 11.29a,

$$\varepsilon = \frac{1-\exp\left[-NTU\left(1-C_r\right)\right]}{1-C_r\exp\left[-NTU\left(1-C_r\right)\right]} \qquad NTU = UA/C_{min} \qquad C_r = C_{min}/C_{max}. \tag{1,2,3}$$

and the outlet temperature is determined from the expression

$$\varepsilon = C_c\left(T_{c,o}-T_{c,i}\right)/C_{min}\left(T_{h,i}-T_{c,i}\right). \tag{4}$$

From Eq. 11.1, the overall heat transfer coefficient is

$$\frac{1}{UA} = \frac{1}{\left(\eta_o hA\right)_h} + \frac{1}{K_t' L} + \frac{1}{\left(\eta_o hA\right)_c} \tag{5}$$

Since circumferential conduction may be significant in the tube walls, η_o needs to be evaluated for each of the tubes.

The convection coefficients are obtained as follows:

Water-side: $Re_D = \frac{4\dot{m}_h}{\pi D \mu} = \frac{4 \times 0.04 \text{ kg/s}}{\pi \times 0.010\text{m} \times 453 \times 10^{-6} \text{ N} \cdot \text{s/m}^2} = 11{,}243.$

The flow is turbulent, and since fully developed, the Dittus-Boelter correlation may be used,

$$\overline{Nu}_h = \bar{h}_h D/k = 0.023 Re_D^{0.8} Pr^{0.3} = 0.023(11{,}243)^{0.8}(2.88)^{0.3} = 54.99$$

$$\bar{h}_h = 54.99 \times 0.656 \text{ W/m} \cdot \text{K}/0.01\text{m} = 3{,}607 \text{ W/m}^2 \cdot \text{K}.$$

Air-side: $Re_D = \frac{4\dot{m}_c}{\pi D \mu} = \frac{4 \times 0.120 \text{ kg/s}}{\pi \times 0.030\text{m} \times 184.6 \times 10^{-7} \text{ N} \cdot \text{s/m}^2} = 275{,}890.$

The flow is turbulent and, since fully developed,

$$\overline{Nu}_c = \bar{h}_c D/K = 0.023 Re_D^{0.8} Pr^{0.4} = 0.023(275{,}890)^{0.8}(0.707)^{0.4} = 450.9$$

$$\bar{h}_c = 450.9 \times 0.0263 \text{ W/m} \cdot \text{K}/0.030\text{m} = 395.3 \text{ W/m}^2 \cdot \text{K}.$$

Water-side temperature effectiveness: $A_h = \pi D_h L = \pi(0.010\text{m})40\text{m} = 1.257\,\text{m}^2$

$$\eta_{o,h} = \eta_{f,h} = \tanh(mL_h)/mL_h \qquad m = \left(\bar{h}_h P/kA\right)^{1/2} = \left(h_h/kt\right)^{1/2}$$

$$m = \left(3607 \text{ W/m}^2 \cdot \text{K}/88 \text{ W/m} \cdot \text{K} \times 0.002\text{m}\right)^{1/2} = 143.2\,\text{m}^{-1}$$

With $L_h = 0.5\,\pi D_h$, $\eta_{o,h} = \tanh(143.2\text{ m}^{-1} \times 0.5\,\pi \times 0.010\text{m})/143.2\text{ m}^{-1} \times 0.5\,\pi \times 0.010\text{ m} =$ 0.435.

Air-side temperature effectiveness: $A_c = \pi D_c L = \pi(0.030\text{m})40\text{m} = 3.770\text{ m}^2$

$$\eta_{o,c} = \eta_{f,c} = \tanh\left(mL_c\right)/mL_c \qquad m = \left(395.3\ \text{W/m}^2\cdot\text{K}/88\ \text{W/m}\cdot\text{K}\times 0.002\text{m}\right)^{1/2} = 47.39\text{ n}$$

With $L_c = 0.5\pi D_c$, $\eta_{o,c} = \tanh(47.39\text{ m}^{-1} \times 0.5\,\pi \times 0.030\text{m})/47.39\text{ m}^{-1} \times 0.5\,\pi \times 0.030\text{m} =$ 0.438.

Hence, from Eq. (5) the UA product is

$$\frac{1}{UA} = \frac{1}{0.435\times 3607\ \text{W/m}^2\cdot\text{K}\times 1.257\,\text{m}^2} + \frac{1}{100\ \text{W/m}\cdot\text{K}\left(40\text{m}\right)} + \frac{1}{0.438\times 395.3\ \text{W/m}^2\cdot\text{K}\times 3.770\,\text{m}^2}$$

$$UA = \left[5.070\times 10^{-4} + 2.50\times 10^{-4} + 1.533\times 10^{-3}\right]^{-1}\text{W/K} = 437\ \text{W/K}.$$

With

$$\left.\begin{array}{l} C_h = \dot{m}_h c_h = 0.040\,\text{kg/s}\times 4186\ \text{J/kg}\cdot\text{K} = 167.4\ \text{W/K} \leftarrow C_{max} \\ C_c = \dot{m}_c c_c = 0.120\,\text{kg/s}\times 1007\ \text{J/kg}\cdot\text{K} = 120.8\ \text{W/K} \leftarrow C_{min} \end{array}\right\} C_r = C_{min}/C_{max} = 0.722$$

$$NTU = \frac{UA}{C_{min}} = \frac{437\ \text{W/K}}{120.8\ \text{W/K}} = 3.62$$

and from Eq. (1) the effectiveness is

$$\varepsilon = \frac{1-\exp\left[-3.62(1-0.722)\right]}{1-0.722\ \exp\left[-3.62(1-0.722)\right]} = 0.862$$

Hence, from Eq. (4), with $C_{min} = C_c$,

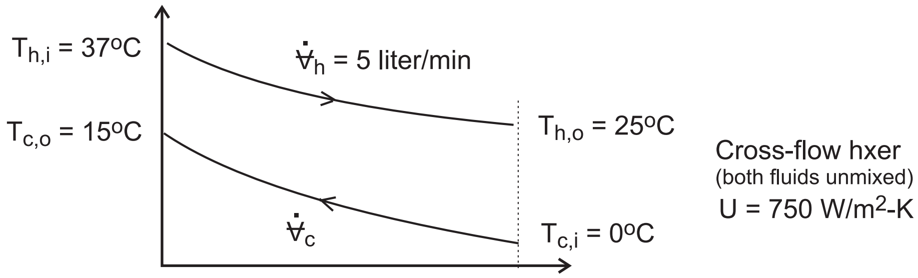
$$0.862 = \frac{C_c\left(T_{c,o} - 23°C\right)}{C_c\left(85-23\right)°C} \qquad T_{c,o} = 76.4°C$$ <

COMMENTS: (1) Using the overall energy balance, the water outlet temperature is

$$T_{h,o} = T_{h,i} + \left(C_c / C_h\right)\left(T_{c,o} - T_{c,i}\right) = 85°C - 0.722\left(76.4-23\right)°C = 46.4°C.$$

(2) To initially evaluate the properties, we assumed that $\overline{T}_h \approx 335$ K and $\overline{T}_c \approx 300$ K. From the calculated values of $T_{h,o}$ and $T_{c,o}$, more appropriate estimates of $\overline{T}_h$ and $\overline{T}_c$ are 338 K and 322 K, respectively. We conclude that proper thermophysical properties were used for water but that the estimates could be improved for air.

Problem 11.34: Use of a cross-flow heat exchanger to cool blood in a cardio-pulmonary bypass procedure.



KNOWN: Cross-flow heat exchanger (both fluids unmixed) cools blood to induce body hypothermia using ice-water as the coolant.

FIND: (a) Heat transfer rate from the blood, (b) Water flow rate, $\dot{\forall}_c$ (liter/min), (c) Surface area of the exchanger, and (d) Calculate and plot the blood and water outlet temperatures as a function of the water flow rate for the range, $2 \le \dot{\forall} \le$ 4 liter/min, assuming all other parameters remain unchanged.

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible losses to the surroundings, (3) Overall heat transfer coefficient remains constant with water flow rate changes, and (4) Constant properties.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_c = 280\text{K}\right)$, $\rho = 1000\ \text{kg/m}^3$, $c = 4198\ \text{J/kg}\cdot\text{K}$.

Blood (given): $\rho = 1050\ \text{kg/m}^3$, $c = 3740\ \text{J/kg}\cdot\text{K}$.

ANALYSIS: (a) The heat transfer rate from the blood is calculated from an energy balance on the hot fluid,

$$\dot{m}_h = \rho_h \dot{\forall}_h = 1050\ \text{kg/m}^3 \times \left(5\ \text{liter/min} \times 1\ \text{min}/60\ \text{s}\right) \times 10^{-3}\ \text{m}^3/\text{liter} = 0.0875\ \text{kg/s}$$

$$q = \dot{m}_h c_h \left(T_{h,i} - T_{h,o}\right) = 0.0875\ \text{kg/s} \times 3740\ \text{J/kg}\cdot\text{K}\left(37 - 25\right)\text{K} = 3927\ \text{W} \qquad < (1)$$

(b) From an energy balance on the cold fluid, find the coolant water flow rate,

$$q = \dot{m}_c\, c_c \left(T_{c,o} - T_{c,i}\right) \qquad (2)$$

$$3927\ \text{W} = \dot{m}_c \times 4198\ \text{J/kg}\cdot\text{K}\left(15 - 0\right)\text{K} \qquad \dot{m}_c = 0.0624\ \text{kg/s}$$

$$\dot{\forall}_c = \dot{m}_c / \rho_c = 0.0624\ \text{kg/s}/1000\ \text{kg/m}^3 \times 10^3\ \text{liter/m}^3 \times 60\ \text{s/min} = 3.74\ \text{liter/min} \qquad <$$

(c) The surface area can be determined using the effectiveness-NTU method. The capacity rates for the exchanger are

$$C_h = \dot{m}_h c_h = 327\ \text{W/K} \qquad C_c = \dot{m}_c\, c_c = 262\ \text{W/K} \qquad C_{min} = C_c \qquad (3, 4, 5)$$

From Eq. 11.18 and 11.19, the maximum heat rate and effectiveness are

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right) = 262\ \text{W/K}\ \left(37 - 0\right)\text{K} = 9694\ \text{W} \qquad (6)$$

$$\varepsilon = q/q_{max} = 3927/9694 = 0.405 \qquad (7)$$

For the cross flow exchanger, with both fluids unmixed, substitute numerical values into Eq. 11.32 to find the number of transfer units, NTU, where $C_r = C_{min}/C_{max}$.

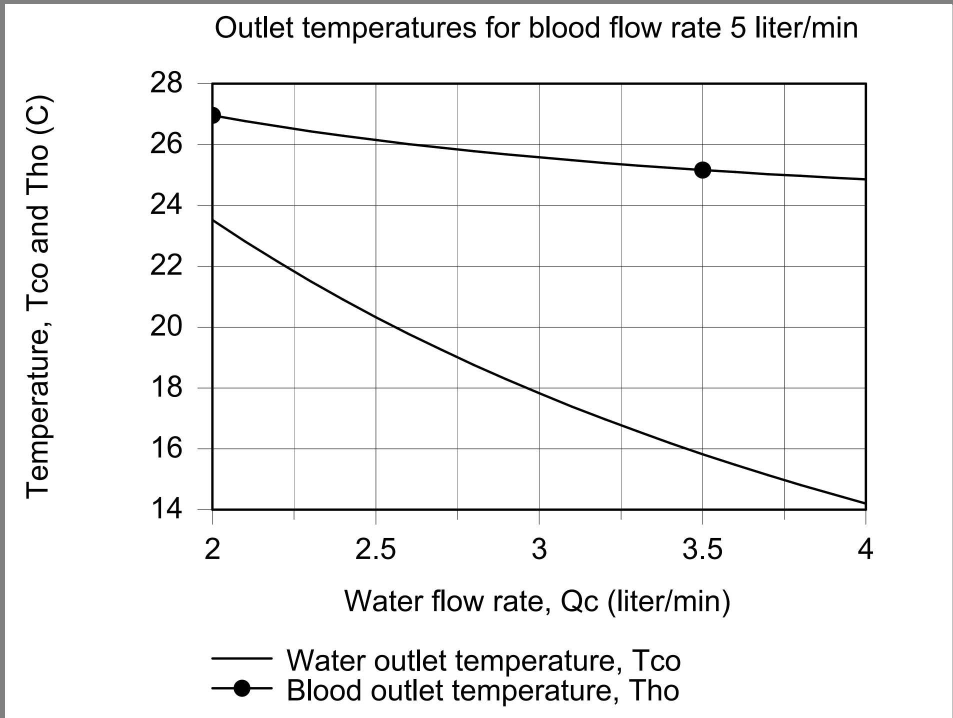
$$\varepsilon = 1 - \exp\left[\left(1/C_r\right)\text{NTU}^{0.22}\left\{\exp\left[-C_r \text{NTU}^{0.78}\right] - 1\right\}\right] \qquad \text{NTU} = 0.691 \qquad (8,9)$$

From Eq. 11.24, find the surface area, A.

$$\text{NTU} = UA/C_{min}$$

$$A = 0.691 \times 262\ \text{W/K}/750\ \text{W/m}^2\cdot\text{K} = 0.241\ \text{m}^2 \qquad <$$

(d) Using the foregoing equations, the blood and water outlet temperatures, $T_{h,o}$ and $T_{c,o}$, respectively, are calculated and plotted as a function of the water flow rate, all other parameters remaining unchanged.



From the graph, note that with increasing water flow rate, both the blood and water outlet temperatures decrease. However, the effect of the water flow rate is greater on the water outlet temperature. This is an advantage for this application, since it is desirable to have the blood outlet temperature relatively insensitive to changes in the water flow rate. That is, if there are pressure changes on the water supply line or a slight mis-setting of the water flow rate controller, the outlet blood temperature will not change markedly.